JOURNAL OF
FUNCTION SPACES AND APPLICATIONS
Volume 5, Number 3 (2007), 299–317

http://www.jfsa.net

The maximal operator in weighted variable spaces $L^{p(\cdot)}$

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(*Communicated by Vladimir D. Stepanov*)

2000 Mathematics Subject Classification. 42B25, 47B38.

Keywords and phrases. Maximal functions, weighted Lebesgue spaces, variable exponent, Carleson curve, Zygmund conditions, Bari-Stechkin class.

Abstract. We study the boundedness of the maximal operator in the weighted spaces $L^{p(\cdot)}(\rho)$ over a bounded open set Ω in the Euclidean space $\mathbb{R}^n$ or a Carleson curve Γ in a complex plane. The weight function may belong to a certain version of a general Muckenhoupt-type condition, which is narrower than the expected Muckenhoupt condition for variable exponent, but coincides with the usual Muckenhoupt class A_p in the case of constant p. In the case of Carleson curves there is also considered another class of weights of radial type of the form $\rho(t) = \prod_{k=1}^m w_k(|t-t_k|)$, $t_k \in \Gamma$, where w_k has the property that $r^{\frac{1}{p(t_k)}} w_k(r) \in \Phi_1^0$, where Φ_1^0 is a certain Zygmund-Bari-Stechkin-type class. It is assumed that the exponent $p(t)$ satisfies the Dini–Lipschitz condition. For such radial type weights the final statement on the boundedness is given in terms of the index numbers of the functions w_k (similar in a sense to the Boyd indices for the Young functions defining Orlich spaces).