

The Riesz “rising sun” lemma for arbitrary Borel measures with some applications

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Abstract. The Riesz “rising sun” lemma is proved for arbitrary locally finite Borel measures on the real line. The result is applied to study an attainability problem of the exact constant in a weak $(1, 1)$ type inequality for the corresponding Hardy-Littlewood maximal operator.

1. Introduction

Let M_+ be the one-sided Hardy-Littlewood maximal operator on the real line

$$M_+f(x) = \sup_{b>x} \frac{1}{b-x} \int_x^b |f| dm, \quad f \in L^1_{\text{loc}}(\mathbb{R}),$$

where m stands for the Lebesgue measure. The following equality

$$(1) \quad m\{M_+f > \lambda\} = \frac{1}{\lambda} \int_{\{M_+f > \lambda\}} |f| dm, \quad \lambda > 0,$$

is well known and sometimes called the Riesz “rising sun lemma” (see [4]) since it can be readily obtained from the following lemma which usually carries this name (see [7], [8]):