

## On dilation operators and sampling numbers

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**Abstract.** We consider the dilation operators  $T_k : f \rightarrow f(2^k \cdot)$  in the frame of Besov spaces  $B_{pq}^s(\mathbb{R}^d)$  with  $1 \leq p, q \leq \infty$ . If  $s > 0$ ,  $T_k$  is a bounded linear operator from  $B_{pq}^s(\mathbb{R}^d)$  into itself and there are optimal bounds for its norm, see [4, 2.3.1]. We study the situation in the case  $s = 0$ , an open problem mentioned also in [4]. It turns out, that new effects based on Littlewood-Paley theory appear.

In the second part of the paper, we apply these results to the study of the so-called *sampling numbers* of the embedding

$$id : B_{pq_1}^{s_1}(\Omega) \rightarrow B_{pq_2}^0(\Omega),$$

where  $\Omega = (0, 1)^d$ . It was observed already in [13] that the estimates from above for the norm of the dilation operator have their immediate counterpart in the estimates from above for the sampling numbers. In this paper we show that even in the limiting case  $s_2 = 0$  (left open so far), this general method supplies optimal results.

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