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A criterion of weak compactness for operators on subspaces of Orlicz spaces

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Abstract. We give a criterion of weak compactness for the operators on the Morse-Transue space M^Ψ, the subspace of the Orlicz space L^Ψ generated by L^∞.

1. Introduction and Notation

In 1975, C. Niculescu established a characterization of weakly compact operators T from $\mathcal{C}(S)$, where S is a compact space, into a Banach space Z ([14, 15], see [3] Theorem 15.2 too): $T\colon \mathcal{C}(S) \to Z$ is weakly compact if and only if there exists a Borel probability measure μ on S such that, for every $\epsilon > 0$, there exists a constant $C(\epsilon) > 0$ such that:

$$\|Tf\| \leq C(\epsilon)\, \|f\|_{L^1(\mu)} + \epsilon\, \|f\|_\infty\,, \quad \forall f \in \mathcal{C}(S).$$

The same kind of result was proved by H. Jarchow for $\mathbb{C}^*$-algebras in [7], and by the first author for $A(\mathbb{D})$ and H^∞ (see [11]). The criterion for H^∞