

The Jensen functional equation in non-Archimedean normed spaces

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Abstract. We investigate the Hyers–Ulam–Rassias stability of the Jensen functional equation in non-Archimedean normed spaces and study its asymptotic behavior in two directions: bounded and unbounded Jensen differences. In particular, we show that a mapping f between non-Archimedean spaces with $f(0) = 0$ is additive if and only if

$$\|f(\frac{x+y}{2}) - \frac{f(x)+f(y)}{2}\| \rightarrow 0$$

as $\max\{\|x\|, \|y\|\} \rightarrow \infty$.

1. Introduction and preliminaries

The history of the stability theory of functional equations started with a problem concerning group homomorphisms posed by S.M. Ulam [30] in 1940 and its solution given by H.D. Hyers [7] in 1941. Hyers' theorem was generalized by T. Aoki [1] for additive mappings and by Th.M. Rassias [24] for linear mappings by considering an unbounded Cauchy difference. The paper [24] of Th.M. Rassias has provided a lot of influence in the development of what we now call Hyers–Ulam–Rassias stability of functional