

A Beurling-Helson type theorem for modulation spaces

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Abstract. We prove a Beurling-Helson type theorem on modulation spaces. More precisely, we show that the only C^1 changes of variables that leave invariant the modulation spaces $\mathcal{M}^{p,q}(\mathbb{R}^d)$ are affine functions on \mathbb{R}^d . A special case of our result involving the Sjöstrand algebra was considered earlier by A. Boulkhemair.

1. Introduction

Given a function ϕ defined from the torus \mathbb{T} to itself, let ϕ^* be the change of variables defined by

$$(1) \quad \phi^*(u) = u \circ \phi$$

for any function u defined on \mathbb{T} .

In 1953, A. Beurling and H. Helson proved that if ϕ is continuous from \mathbb{T} into itself and if ϕ^* is a bounded linear operator on the Fourier algebra $A(\mathbb{T}) = A_1(\mathbb{T})$ of absolutely convergent Fourier series, then necessarily $\phi(t) = kt + \phi(0)$ for some $k \in \mathbb{Z}$ [1]. The proof of this result involved