

**Parameter depending almost monotonic functions
and their applications to dimensions in
metric measure spaces**

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Abstract. In connection with application to various problems of operator theory, we study almost monotonic functions $w(x, r)$ depending on a parameter x which runs a metric measure space X , and the so called index numbers $m(w, x), M(w, x)$ of such functions, and consider some generalized Zygmund, Bary, Lozinskii and Stechkin conditions. The main results contain necessary and sufficient conditions, in terms of lower and upper bounds of indices $m(w, x)$ and $M(w, x)$, for the uniform belongness of functions $w(\cdot, r)$ to Zygmund-Bary-Steckin classes.

We give also applications to local dimensions in metric measure spaces and characterization of some integral inequalities involving radial weights and measures of balls in such spaces.

1. Introduction

Last decades there was observed an increasing interest to the study of function spaces whose characteristics may vary from point to point. A well known typical example is the generalized Lebesgue space $L^{p(\cdot)}$ with variable exponent, see for instance the surveying papers [2], [11], [25] on harmonic