

Radial variation in some function spaces

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Abstract. In a previous paper [8] we considered properties of the radial variation of analytic functions in a class of Besov spaces A_{pq}^s , $s > 0$. Here we wish to extend these results to certain related spaces. These are the Lipschitz classes Λ_s and the mean Lipschitz classes $\Lambda_{p,s}$ where $p \geq 1, 0 < s < 1$. We also consider A_{pq}^0 , where $s = 0$, although the results obtained for these are not as good as when $s > 0$.

1. Introduction

If f is analytic in the disc, the radial variation function of f is the function defined on the disc by

$$(1) \quad F(r, t) = \int_0^r |f'(ue^{it})| du, \quad r < 1, \quad 0 \leq t \leq 2\pi.$$

Since $f(re^{it}) - f(0) = \int_0^r f'(ue^{it}) du$, it is clear that

$$|f(re^{it})| \leq |f(0)| + F(r, t), \quad r < 1, \quad 0 \leq t \leq 2\pi,$$