

Some new refinements of strengthened Hardy and Pólya–Knopp’s inequalities

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Abstract. We prove a new general one-dimensional inequality for convex functions and Hardy–Littlewood averages. Furthermore, we apply this result to unify and refine the so-called Boas’s inequality and the strengthened inequalities of the Hardy–Knopp–type, deriving their new refinements as special cases of the obtained general relation. In particular, we get new refinements of strengthened versions of the well-known Hardy and Pólya–Knopp’s inequalities.

1. Introduction

To begin with, we recall some well-known classical integral inequalities. If $p > 1$, $k \neq 1$, and the function F is defined on $\mathbb{R}_+ = \langle 0, \infty \rangle$ by

$$F(x) = \begin{cases} \int_0^x f(t) dt, & k > 1, \\ \int_x^\infty f(t) dt, & k < 1, \end{cases}$$