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Endpoint estimates for homogeneous Littlewood-Paley g-functions with non-doubling measures

Dachun Yang and Dongyong Yang

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Abstract. Let μ be a nonnegative Radon measure on $\mathbb{R}^d$ which satisfies the growth condition that there exist constants $C_0 > 0$ and $n \in (0, d]$ such that for all $x \in \mathbb{R}^d$ and $r > 0$, $\mu(B(x, r)) \le C_0 r^n$, where $B(x, r)$ is the open ball centered at x and having radius r. In this paper, when $\mathbb{R}^d$ is not an initial cube which implies $\mu(\mathbb{R}^d) = \infty$, the authors prove that the homogeneous Littlewood-Paley g-function of Tolsa is bounded from the Hardy space $H^1(\mu)$ to $L^1(\mu)$, and furthermore, that if $f \in \mathrm{RBMO}\,(\mu)$, then $[\dot{g}(f)]^2$ is either infinite everywhere or finite almost everywhere, and in the latter case, $[\dot{g}(f)]^2$ belongs to $\mathrm{RBLO}\,(\mu)$ with norm no more than $C\|f\|^2_{\mathrm{RBMO}\,(\mu)}$, where $C > 0$ is independent of f.

1. Introduction

Recall that a *non-doubling measure* μ on $\mathbb{R}^d$ means that μ is a nonnegative Radon measure which only satisfies the following growth