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On small Lebesgue spaces

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Abstract. We consider a generalized version of the small Lebesgue spaces, introduced in [5] as the associate spaces of the grand Lebesgue spaces. We find a simplified expression for the norm, prove relevant properties, compute the fundamental function and discuss the comparison with the Orlicz spaces.

1. Introduction

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain and $f : \Omega \to \mathbb{R}^N$, $f = (f^1, \ldots, f^N)$ be a mapping of Sobolev class $W^{1,N}_{loc}(\Omega, \mathbb{R}^N)$. Let us denote by $Df(x) : \mathbb{R}^N \to \mathbb{R}^N$ the differential and by $J(x, f) = \det Df(x)$ the Jacobian of f. After the elementary remark that by Hölder's inequality the Jacobian $J(x, f)$ is in $L^1_{loc}(\Omega)$, the first fundamental result on the integrability of the Jacobian was due to Müller ([24]). He proved that if f is an orientation preserving mapping, i.e. $J(x, f) \geq 0$ for a.e. $x \in \Omega$, then J enjoys a greater degree of integrability, namely, it belongs locally to the Zygmund class $L \log L$. As a dual result, Iwaniec and Sbordone in [19] introduced the grand Lebesgue space $L^{N)}(\Omega)$, and proved that under the assumption