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Two-scale convergence with respect to measures and homogenization of monotone operators

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Abstract. In 1989 Nguetseng introduced two-scale convergence, which now is a frequently used tool in homogenization of partial differential operators. In this paper we discuss the notion of two-scale convergence with respect to measures. We make an exposition of the basic facts of this theory and develope it in various ways. In particular, we consider both variable L^p spaces and variable Sobolev spaces. Moreover, we apply the results to a homogenization problem connected to a class of monotone operators.

1. Introduction

Let Ω be a bounded open subset of $\mathbb{R}^N$, where the Lebesgue measure of the boundary is zero, $Y = [0,1)^N$ the semi-open cube in $\mathbb{R}^N$ and (ε) a sequence of positive numbers converging to 0. In 1989 G. Nguetseng, see [7], proved that for each bounded sequence (u_ε) in $L^2(\Omega)$ there exists a subsequence, still indexed by ε, and a $u \in L^2(\Omega \times Y)$ such that

(1) $$\int_\Omega u_\varepsilon(x)\phi\left(x, \frac{x}{\varepsilon}\right)\,dx \to \int_\Omega \int_Y u(x,y)\phi(x,y)\,dydx\,,$$