

## Sobolev type inequalities in ultrasymmetric spaces with applications to Orlicz-Sobolev embeddings

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**Abstract.** Let  $D^k f$  mean the vector composed by all partial derivatives of order  $k$  of a function  $f(x)$ ,  $x \in \Omega \subset \mathbb{R}^n$ . Given a Banach function space  $A$ , we look for a possibly small space  $B$  such that  $\|f\|_B \leq c \| |D^k f| \|_A$  for all  $f \in C_0^k(\Omega)$ . The estimates obtained are applied to ultrasymmetric spaces  $A = L_{\varphi, E}$ ,  $B = L_{\psi, E}$ , giving some optimal (or rather sharp) relations between parameter-functions  $\varphi(t)$  and  $\psi(t)$  and new results for embeddings of Orlicz-Sobolev spaces.

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### 1. Introduction

Let  $W_p^k = W_p^k(\Omega)$  mean a standard Sobolev space on the bounded open set  $\Omega \subset \mathbb{R}^n$  (for simplicity, we may assume  $|\Omega| = 1$ ). As known, the classical Sobolev embedding

$$(1.1) \quad W_p^k \hookrightarrow L_q, \quad 1 < p < n/k, \quad q = np/(n - kp)$$