JOURNAL OF
FUNCTION SPACES AND APPLICATIONS
Volume 3, Number 2 (2005), 117-124



# An inequality for first-order differences*

**Gord Sinnamon**

(*Communicated by Victor Burenkov*)

**2000 Mathematics Subject Classification.** 26D10.
**Keywords and phrases.** Differences, integral inequalities.

---

**Abstract.** A question about comparing norms of difference operators that was raised in [1] and presented at the Fourth ISAAC Congress is answered in the affirmative.

---

Let $1 \le p < \infty$ and suppose $f : [0, \infty) \to \mathbf{R}$. For $h > 0$, set

$$F(h) \equiv \|\Delta_h f\|_{L^p(0,h)} = \left( \int_0^h |f(x+h) - f(x)|^p \, dx \right)^{1/p}$$

and

$$G(h) \equiv \|\Delta_h f\|_{L^p(h,3h)} = \left( \int_h^{3h} |f(x+h) - f(x)|^p \, dx \right)^{1/p}.$$

We wish to compare $F$ and $G$ as functions of $h$. It is easy to see that pointwise comparisons of $F$ and $G$ are impossible for arbitrary $f$ as $F$ depends on values of $f$ on the interval $[0, 2h]$ but $G$ depends on values of

*Supported by the Natural Sciences and Engineering Research Council of Canada.