

## The random Wigner distribution of Gaussian stochastic processes with covariance in $S_0(\mathbb{R}^{2d})$

Patrik Wahlberg

*(Communicated by Hans Feichtinger)*

**2000 Mathematics Subject Classification.** 42B35, 60G15, 60G35, 62M15, 94A12.

**Keywords and phrases.** Time-frequency analysis, the Feichtinger algebra, Gaussian scalar-valued stochastic processes, Wigner distribution, Wigner spectrum, Cohen's class.

---

**Abstract.** The paper treats time-frequency analysis of scalar-valued zero mean Gaussian stochastic processes on  $\mathbb{R}^d$ . We prove that if the covariance function belongs to the Feichtinger algebra  $S_0(\mathbb{R}^{2d})$  then: (i) the Wigner distribution and the ambiguity function of the process exist as finite variance stochastic Riemann integrals, each of which defines a stochastic process on  $\mathbb{R}^{2d}$ , (ii) these stochastic processes on  $\mathbb{R}^{2d}$  are Fourier transform pairs in a certain sense, and (iii) Cohen's class, ie convolution of the Wigner process by a deterministic function  $\Phi \in C(\mathbb{R}^{2d})$ , gives a finite variance process, and if  $\Phi \in S_0(\mathbb{R}^{2d})$  then  $W * \Phi$  can be expressed multiplicatively in the Fourier domain.

---

### 1. Introduction

For functions and distributions defined on  $\mathbb{R}^d$  it is well known under what circumstances it is possible to define the Wigner distribution and the ambiguity function [7]. This paper concerns the corresponding conditions