

Characterizations of inner product spaces by orthogonal vectors

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Abstract. Let X be a real normed space with unit closed ball B . We prove that X is an inner product space if and only if it is true that whenever x, y are points in ∂B such that the line through x and y supports $\frac{\sqrt{2}}{2}B$ then $x \perp y$ in the sense of Birkhoff.

1. Introduction

Several known characterizations of inner product spaces are available in the literature [1]. This paper concerns a new characterization by means of orthogonal vectors. Let X be a real normed space (of dimension ≥ 2) and let S_X denote its unit sphere. If $x, y \in S_X$, we set:

$$k(x, y) = \inf\{\|tx + (1-t)y\| : t \in [0, 1]\}$$

moreover if $x, y \in S_X$ then $x \perp y$ denotes the orthogonality in the sense of Birkhoff [1] i.e. $\|x\| \leq \|x + \lambda y\| \quad \forall \lambda \in \mathbb{R}$. In this paper we consider