

## New proof of Nagnibida's theorem

Mubariz T. Karaev

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**Abstract.** Using the Duhamel product for holomorphic functions we give a new proof of Nagnibida's theorem on unicellularity of integration operator  $J_\alpha, (J_\alpha f)(z) = \int_\alpha^z f(t)dt$ , acting in the space  $\mathcal{Hol}(\Omega)$ .

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Let  $\alpha$  be a fixed complex number,  $\Omega$  be a simple-connected domain in the plane containing the number  $\alpha$ , and assume that  $\Omega$  is star-shaped with respect to the  $\alpha$  in the sense that  $z \in \Omega$  implies  $\lambda z + (1 - \lambda)\alpha \in \Omega$  for each  $\lambda \in (0, 1)$ . Let  $\mathcal{Hol}(\Omega)$  be the vector space of all functions holomorphic in  $\Omega$  with the topology of compact convergence, and let  $J_\alpha$  denotes the integration operator acting in  $\mathcal{Hol}(\Omega)$  by formula

$$(J_\alpha f)(z) = \int_\alpha^z f(t)dt,$$

where integral is taken on the line segment with the ends  $\alpha$  and  $z$ .