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Weighted norm inequalities and indices

Joaquim Martín* **and Mario Milman**

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Abstract. We extend and simplify several classical results on weighted norm inequalities for classical operators acting on rearrangement invariant spaces using the theory of indices. As an application we obtain necessary and sufficient conditions for generalized Hardy type operators to be bounded on $\Lambda_p(w)$, $\Lambda_{p,\infty}(w)$, $\Gamma_p(w)$ and $\Gamma_{p,\infty}(w)$.

1. Introduction

Ariño and Muckenhoupt [2] characterized the class of weights B_p for which the Hardy-Littlewood maximal operator is bounded on classical Lorentz spaces $\Lambda_p(w)$ defined by $f \in \Lambda_p(w) \Leftrightarrow \|f\|_{\Lambda_p(w)} < \infty$, where the quasi norm $\|f\|_{\Lambda_p(w)}$ is given by

$$\|f\|_{\Lambda_p(w)} = \|f^* w^{1/p}\|_{L^p(0,\infty)} = \|f^*\|_{L^p(w)} \tag{1.1}$$

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