

Three weights higher order Hardy type inequalities

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(Communicated by Vladimir Stepanov)

2000 Mathematics Subject Classification. 26D10, 26D15.

Keywords and phrases. Inequalities, higher order Hardy type inequalities, kernels, weights, weighted differential operator.

Abstract. We investigate the following three weights higher order Hardy type inequality

$$(0.1) \quad \|g\|_{q,u} \leq C \|D_{\rho}^k g\|_{p,v},$$

where D_{ρ}^i denotes the following weighted differential operator:

$$D_{\rho}^i g(t) = \begin{cases} \frac{d^i g(t)}{dt^i}, & i = 0, 1, \dots, m-1, \\ \frac{d^{i-m}}{dt^{i-m}} \left(\rho(t) \frac{d^m g(t)}{dt^m} \right), & i = m, m+1, \dots, k, \end{cases}$$

for a weight function $\rho(\cdot)$.

A complete description of the weights u , v and ρ so that (0.1) holds was given in [4] for the case $1 < p \leq q < \infty$. Here the corresponding characterization is proved for the case $1 < q < p < \infty$. The crucial step in the proof of the main result is to use a new Hardy type inequality (for a Volterra type operator), which we first state and prove.
