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Nonlinear eigenvalue problems in Sobolev spaces with variable exponent

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Abstract. We study the boundary value problem $-\mathrm{div}((|\nabla u|^{p_1(x)-2} + |\nabla u|^{p_2(x)-2})\nabla u) = f(x,u)$ in Ω, $u = 0$ on $\partial\Omega$, where Ω is a smooth bounded domain in $\mathbb{R}^N$. We focus on the cases when $f_\pm(x,u) = \pm(-\lambda|u|^{m(x)-2}u + |u|^{q(x)-2}u)$, where $m(x) := \max\{p_1(x), p_2(x)\} < q(x) < \dfrac{N \cdot m(x)}{N - m(x)}$ for any $x \in \overline{\Omega}$. In the first case we show the existence of infinitely many weak solutions for any $\lambda > 0$. In the second case we prove that if λ is large enough then there exists a nontrivial weak solution. Our approach relies on the variable exponent theory of generalized Lebesgue-Sobolev spaces, combined with a $\mathbb{Z}_2$-symmetric version for even functionals of the Mountain Pass Lemma and some adequate variational methods.

1. Introduction and preliminary results

Electrorheological fluids (sometimes referred to as "smart fluids"), are particular fluids of high technological interest whose apparent viscosity