JOURNAL OF
FUNCTION SPACES AND APPLICATIONS
Volume 4, Number 3 (2006), 329-341

http://www.jfsa.net

Modulation spaces $M^{p,q}$ for $0 < p,\ q \leqq \infty$

Masaharu Kobayashi

(*Communicated by Hans Triebel*)

2000 Mathematics Subject Classification. 42B35, 46A16, 46E30, 46E35.

Keywords and phrases. Modulation spaces, quasi-Banach spaces, L^p estimates for convolutions, density of the rapidly decreasing functions.

Abstract. The purpose of this paper is to construct modulation spaces $M^{p,q}(\mathbf{R}^d)$ for general $0 < p, q \leqq \infty$, which coincide with the usual modulation spaces when $1 \leqq p, q \leqq \infty$, and study their basic properties including their completeness. Given any $g \in \mathcal{S}(\mathbf{R}^d)$ such that supp $\widehat{g} \subset \{\xi \mid |\xi| \leqq 1\}$ and $\sum_{k \in \mathbf{Z}^d} \widehat{g}(\xi - \alpha k) \equiv 1$, our modulation space consists of all tempered distributions f such that the (quasi)-norm

$$\|f\|_{M^{p,q}_{[g]}} := \left(\int_{\mathbf{R}^d} \left(\int_{\mathbf{R}^d} \left| f * (M_\omega g)(x) \right|^p dx \right)^{\frac{q}{p}} d\omega \right)^{\frac{1}{q}}$$

is finite.

1. Introduction

The purpose of this paper is to construct modulation spaces $M^{p,q}(\mathbf{R}^d)$ for general $0 < p, q \leqq \infty$, which coincide with the usual modulation spaces when $1 \leqq p, q \leqq \infty$, and study their basic properties.