

Dual of modulation spaces

Masaharu Kobayashi

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Abstract. The aim of this paper is the study of the dual of modulation spaces $M^{p,q}(\mathbf{R}^d)$ for $0 < p, q < \infty$.

1. Introduction

We have constructed the modulation spaces $M^{p,q}(\mathbf{R}^d)$ in [2] for general $0 < p, q \leq \infty$, which coincide with the usual modulation spaces when $1 \leq p, q \leq \infty$, and studied their basic properties. The aim of this paper is the study of the dual of $M^{p,q}(\mathbf{R}^d)$ for $0 < p, q < \infty$. When $1 \leq p, q < \infty$, the fact that $M^{p',q'}(\mathbf{R}^d)$ is the dual of $M^{p,q}(\mathbf{R}^d)$ is already known, where $1/p + 1/p' = 1/q + 1/q' = 1$. (See Feichtinger [1].) So in this paper we are concerned with the dual, in particular when $p < 1$ or $q < 1$. Motivated by the fact that the modulation spaces have similar properties to that of the Besov spaces (Proposition 2.2), we employ H. Triebel's method [3] to study the dual. But gained results are similar to the sequence spaces l^p rather than the Besov spaces $B_{p,q}^s(\mathbf{R}^d)$.