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A note on maximal operator on $\ell^{\{p_n\}}$ and $L^{p(x)}(\mathbb{R})$

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Abstract. We consider a discrete analogue of Hardy-Littlewood maximal operator on the generalized Lebesque space $\ell^{\{p_n\}}$ of sequences defined on $\mathbb{Z}$. It is known a necessary and sufficient condition $\mathcal{P}$ which guarantees an existence of a real number $p > 1$ such that the norms in the space $\ell^{\{p_n\}}$ and in the classical space ℓ^p are equivalent. Of course, this condition immediately implies the boundedness of maximal operator on $\ell^{\{p_n\}}$ and, moreover, $\lim_{|n|\to\infty} p_n = p$. We construct two examples of sequences $\{p_n\}$ satisfying $\lim_{|n|\to\infty} p_n = p$ in this paper. In the first example the maximal operator is unbounded on $\ell^{\{p_n\}}$ and the sequence $\{p_n\}$ from the second example does not satisfy $\mathcal{P}$ but the maximal operator is bounded. Moreover, it is known a sufficient integral condition to a behavior of a function $p(x)$ at infinity which guarantees the boundedness of the maximal operator on $L^{p(\cdot)}(\mathbb{R}^n)$. As a main result of this paper we construct a function $p(x)$ which does not satisfy this integral condition nevertheless the maximal operator is bounded.

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