

A homogeneity property for Besov spaces

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Abstract. A homogeneity property for some Besov spaces $B_{p,q}^s$ is proved. An analogous property for some $F_{p,q}^s$ spaces is already known.

1. Introduction

In the present note we prove the so-called *homogeneity property* for some Besov spaces on \mathbb{R}^n . We show that for $0 < p, q \leq \infty$ and $s > \sigma_p$,

$$(1) \quad \|f(\lambda \cdot) | B_{p,q}^s(\mathbb{R}^n)\| \sim \lambda^{s-\frac{n}{p}} \|f | B_{p,q}^s(\mathbb{R}^n)\|,$$

for all $0 < \lambda \leq 1$ and all

$$(2) \quad f \in B_{p,q}^s(\mathbb{R}^n) \quad \text{with} \quad \text{supp } f \subset \{x \in \mathbb{R}^n : |x| \leq \lambda\}.$$

A corresponding property for the spaces $F_{p,q}^s(\mathbb{R}^n)$ may be found in [9, Corollary 5.16, p. 66]. It is of some use in connection with refined localization, non-smooth atoms, scaling properties and pointwise multipliers. We refer to [10, Section 4.2.2]. Here we prove this property for the spaces $B_{p,q}^s(\mathbb{R}^n)$, with $s > \sigma_p$. For the particular case where $\sigma_p < s < n/p$ this property can be obtained via atomic decompositions. This was the starting point for the main result presented in this note.