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Bounds on the effective behavior of a homogenized generalized Reynolds equation

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Abstract. We study upper and lower bounds for estimating the effective behavior described by homogenizing a problem which is a generalization of the Reynold equation. All cases when these bounds coincide are also found.

1. Introduction

We consider an equation of the form

(1)
$$\frac{\partial}{\partial x_1}\left(a_1\left(x,\frac{x}{\varepsilon}\right)\frac{\partial u_\varepsilon}{\partial x_1}-b_1\left(x,\frac{x}{\varepsilon}\right)\right)+\frac{\partial}{\partial x_2}\left(a_2\left(x,\frac{x}{\varepsilon}\right)\frac{\partial u_\varepsilon}{\partial x_2}-b_2\left(x,\frac{x}{\varepsilon}\right)\right)=f(x),$$

$x \in \Omega \subset R^2$, $u_\varepsilon \in H_0^1(\Omega)$. Here, a_i and b_i are assumed to be piecewise continuous in the first variable and measurable and periodic relative to a cell $Y = [0,1]^2$ in the second variable. In addition, we assume that there exist constants k_- and k_+ such that

$$0 < k_- \le a_i(x,y) \le k_+ < \infty \quad \text{and} \quad |b_i(x,y)| \le k_+$$