JOURNAL OF
FUNCTION SPACES AND APPLICATIONS
Volume 5, Number 2 (2007), 183-198

http://www.jfsa.net

A direct proof of Sobolev embeddings for quasi-homogeneous Lizorkin–Triebel spaces with mixed norms

Jon Johnsen and Winfried Sickel

(*Communicated by Jürgen Appell*)

2000 Mathematics Subject Classification. 46E35.

Keywords and phrases. Function spaces of Besov and Lizorkin–Triebel type, anisotropic spaces, mixed norms, Sobolev embeddings, geometric rectangle condition.

Abstract. The article deals with a simplified proof of the Sobolev embedding theorem for Lizorkin–Triebel spaces (that contain the L_p-Sobolev spaces H_p^s as special cases). The method extends to a proof of the corresponding fact for general Lizorkin–Triebel spaces based on mixed L_p-norms. In this context a Nikol′skij–Plancherel–Polya inequality for sequences of functions satisfying a geometric rectangle condition is proved. The results extend also to anisotropic spaces of the quasi-homogeneous type.

1. Introduction

To give an overview, we first comment on standard Lizorkin–Triebel spaces (i.e. isotropic, inhomogeneous spaces with unmixed norms). These are throughout denoted by $F_{p,q}^s$.

Since around 1977 the question of Sobolev embeddings of Lizorkin–Triebel spaces has been answered affirmatively, with a unified proof of the following