

On composition operators in Q_K type spaces

Marko Kotilainen

(Communicated by Miroslav Engliš)

2000 Mathematics Subject Classification. 47B33, 30D50.

Keywords and phrases. Composition operator, $Q_K(p, q)$, Q_K spaces, the Bloch space.

Abstract. Let $p \geq 1$, $q > -2$ and let $K : [0, \infty) \rightarrow [0, \infty)$ be nondecreasing. With a different choice of p , q and K , the Banach space $Q_K(p, q)$ coincides with many well-known analytic function spaces. Boundedness and compactness of the composition operator C_φ from α -Bloch space \mathcal{B}^α into $Q_K(p, q)$ are characterized by a condition depending only on analytic mapping $\varphi : \mathbb{D} \rightarrow \mathbb{D}$. The same properties are also studied in the case $C_\varphi : Q_K(p, q) \rightarrow \mathcal{B}^\alpha$.

1. Introduction

The space $Q_K(p, q)$ was introduced in [17]. To recall the definition, let \mathbb{D} denote the unit disc and let $\sigma_a(z) = (a - z)/(1 - \bar{a}z)$ be the automorphism in \mathbb{D} , which interchanges points 0 and a . Then $g(z, a) = -\log |\sigma_a(z)|$ is the Green function. Let $p > 0$, $q > -2$ and $K : [0, \infty) \rightarrow [0, \infty)$ be right-continuous and nondecreasing. An analytic function f in \mathbb{D} belongs to $Q_K(p, q)$ if

$$\|f\|_{Q_K(p, q)}^p = \sup_{a \in \mathbb{D}} \int_{\mathbb{D}} |f'(z)|^p (1 - |z|^2)^q K[g(z, a)] dA(z) < \infty.$$