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Extremal points without compactness in $L^1(\mu)$

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Abstract. We investigate the existence of extremal points and the Krein-MIlman representation $A = \overline{\text{co}}\,\text{Ext}A$ of bounded convex subsets of $L^1(\mu)$ which are only closed with respect to the topology of μ-a.e. convergence.

1. Introduction

As shown in [2] convex bounded substets of $L^1(\mu)$ that are closed with respect to the topology of μ-a.e. convergence enjoy several properties that are usually a consequence of the compactness in other vector topologies: Optimization without compactness, separation, Mazur's convergence.

In this paper we show that these properties also include the existence of extremal points (**Theorem 5** below), provided the set is bounded from below. We also investigate bounded, closed, convex subsets $A \subset L^1(\mu)$ that fulfill the Krein-Milman representation $A = \overline{\text{co}}\,\text{Ext}A$, and compare the result versus weak compactness of A; more precisely we provide an example of a μ-closed, convex set bounded from below that is not weakly compact.