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On the boundedness of operators in $L^p(l^q)$ and Triebel-Lizorkin Spaces

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Abstract. Given a bounded linear operator $T : L^{p_0}(\mathbb{R}^n) \to L^{p_1}(\mathbb{R}^n)$, for $1 \leq p_0, p_1 \leq \infty$, we state conditions under which T defines a bounded operator between corresponding pairs of $L^p(\mathbb{R}^n; l^q)$ spaces and Triebel-Lizorkin spaces $F^s_{p,q}(\mathbb{R}^n)$. Applications are given to linear parabolic equations and to Schrödinger semigroups.

1. Introduction

Let $1 \leq p \leq \infty$. We define the linear spaces $L^p(\mathbb{R}^n; l^q) = L^p(l^q)$ as the set of all sequences $\{u_k\}_{k\in\mathbb{N}_0}$ of complex-valued measurable functions in $\mathbb{R}^n$ for which

$$\|\{u_k\}\|_{L^p(l^q)} = \begin{cases} \|(\sum_{k=0}^{\infty} |u_k|^q)^{1/q}\|_{L^p} & 1 \leq q < \infty \\ \|\sup_{k\geq 0}(|u_k|)\|_{L^p} & q = \infty \end{cases}$$

is finite. Each of the spaces $L^p(\mathbb{R}^n; l^q)$ is a Banach space with the norm $\|\cdot\|_{L^p(l^q)}$ and L^p is (isometrically) embedded in $L^p(l^q)$ via the map

$$u \mapsto \{\delta_{0k} u\}_{k\in\mathbb{N}_0},$$

where δ_{0k} is the Kronecker symbol: $\delta_{0k} = 1$ if $k = 0$ and $\delta_{0k} = 0$ if $k \neq 0$.