

Convolution on spaces of locally summable functions

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Abstract. In this work we prove the existence of convolution on Marcinkiewicz spaces $\mathfrak{M}^p(\mathbb{R})$, $1 \leq p < \infty$, and, using pointwise approximate identities, we extend the classical definition of Hilbert transform to such spaces.

1. Introduction

We are interested in extending classical results in the theory of singular integrals to spaces of functions that are only locally summable. The natural environment should be the space $L^p_{\text{loc}}(\mathbb{R})$, but the most typical singular integral, the Hilbert transform, does not converge therein. Therefore we need some decay condition. An interesting way to introduce mild decay conditions is based upon boundedness of integral averages over large intervals in \mathbb{R} . This leads to Marcinkiewicz spaces $\mathfrak{M}^p(\mathbb{R})$ defined in Section 2. These spaces have been studied also in [3] and in [1].

In [1], the author proves that any regular bounded Borel measure gives rise to a bounded convolution operator on \mathfrak{M}^p , for $p \geq 1$.