

Spaces of Sobolev type with positive smoothness on \mathbb{R}^n , embeddings and growth envelopes

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(Communicated by Hans Triebel)

2000 Mathematics Subject Classification. 46E35.

Keywords and phrases. Triebel-Lizorkin spaces, Besov spaces, Sobolev spaces, embeddings, growth envelopes.

Abstract. We characterize Triebel-Lizorkin spaces with positive smoothness on \mathbb{R}^n , obtained by different approaches. First we present three settings $\mathbf{F}_{p,q}^s(\mathbb{R}^n)$, $F_{p,q}^s(\mathbb{R}^n)$, $\mathfrak{F}_{p,q}^s(\mathbb{R}^n)$ associated to definitions by differences, Fourier-analytical methods and subatomic decompositions. We study their connections and diversity, as well as embeddings between these spaces and into Lorentz spaces. Secondly, relying on previous results obtained for Besov spaces $\mathfrak{B}_{p,q}^s(\mathbb{R}^n)$, we determine their growth envelopes $\mathfrak{E}_G(\mathfrak{F}_{p,q}^s(\mathbb{R}^n))$ for $0 < p < \infty$, $0 < q \leq \infty$, $s > 0$, and finally discuss some applications.

1. Introduction

In this article Triebel-Lizorkin spaces of positive smoothness on \mathbb{R}^n are investigated. They were introduced independently by Triebel and Lizorkin in the early 1970s. For a detailed treatment together with historical remarks we refer to Triebel [18, 19]. The idea for this paper originates from its forerunner [12], where we studied corresponding problems for Besov spaces. Since the substantial theory of the Triebel-Lizorkin spaces is strongly linked