

**The boundedness of commutator of Riesz transform  
associated with Schrödinger operators  
on a Hardy space**

Canqin Tang<sup>1</sup> and Chuanmei Bi  
(*Communicated by Vladimir Maz'ya*)

**2000 Mathematics Subject Classification.** Primary: 42B20, 42B30.

**Keywords and phrases.** Commutator, *BMO*, Riesz transform, Schrödinger operators, Hardy space.

---

**Abstract.** In this paper, we study the boundedness of commutator  $[b, T]$  of Riesz transform associated with Schrödinger operator and  $b$  is *BMO* type function, note that the kernel of  $T$  has no smoothness, and the boundedness from  $H_b^1(\mathbb{R}^n) \rightarrow L^1(\mathbb{R}^n)$  is obtained.

---

## 1. Introduction

It is well known that the Calderón-Zygmund singular operator is an important operator in Harmonic Analysis. The properties of the C-Z singular operator and its commutator are studied by many scholars. Such as in [1] [2] [3]. Among this, C. Perez [3] states the  $H_b^1(\mathbb{R}^n) \rightarrow L^1(\mathbb{R}^n)$  boundedness of the commutator  $[b, T]$ , where  $T$  is a C-Z singular operator and  $b \in BMO(\mathbb{R}^n)$ .

Schrödinger differential operator is another interesting topic in Harmonic Analysis. Let  $A = -\Delta + V(x)$  be the Schrödinger differential operator

---

<sup>1</sup>This work is partially supported by NNSF 107010178