JOURNAL OF
FUNCTION SPACES AND APPLICATIONS
Volume 7, Number 3 (2009), 289–300

On Bellman-Golubov theorems for the Riemann-Liouville operators

Pham Tien Zung

(*Communicated by Victor Burenkov*)

2000 Mathematics Subject Classification. Primary 26D10; Secondary 26D15, 26D07.

Keywords and phrases. Fourier integral, Lebesgue spaces, Riemann-Liouville operator.

Abstract. Superposition of Fourier transform with the Riemann - Liouville operators is studied.

1. Introduction

Let $\mathbb{R} := (-\infty, +\infty)$. We denote $\|f\|_p := \left(\int_{\mathbb{R}} |f(x)|^p dx\right)^{1/p}$ for $1 \le p < \infty$ and $\|f\|_\infty := \text{esssup}_{x\in\mathbb{R}}|f(x)|$. By $L^p(\mathbb{R})$ we denote the Lebesgue space of all measurable functions on $\mathbb{R}$ such that $\|f\|_p < \infty$. Similar notations are applied for $\mathbb{R}_+ := [0, +\infty)$.

For $f \in L^1(\mathbb{R})$, the Fourier transform Ff is defined by

$$Ff(x) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{ixt}dt.$$

In particular cases, when f is even or odd, the Fourier transforms are

$$F_c f(x) := \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t)\cos xt dt$$