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Marcinkiewicz integrals with variable kernels on Hardy and weak Hardy spaces*

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Abstract. In this article, we consider the Marcinkiewicz integrals with variable kernels defined by

$$\mu_\Omega(f)(x) = \left(\int_0^\infty \left| \int_{|x-y|\le t} \frac{\Omega(x, x-y)}{|x-y|^{n-1}} f(y)dy \right|^2 \frac{dt}{t^3} \right)^{1/2},$$

where $\Omega(x,z) \in L^\infty(\mathbb{R}^n) \times L^q(\mathbb{S}^{n-1})$ for $q > 1$. We prove that the operator μ_Ω is bounded from Hardy space, $H^p(\mathbb{R}^n)$, to $L^p(\mathbb{R}^n)$ space; and is bounded from weak Hardy space, $H^{p,\infty}(\mathbb{R}^n)$, to weak $L^p(\mathbb{R}^n)$ space for $\max\{\frac{2n}{2n+1}, \frac{n}{n+\alpha}\} < p < 1$, if Ω satisfies the $L^{1,\alpha}$-Dini condition with any $0 < \alpha \le 1$.

1. Introduction

Let $\mathbb{R}^n (n \ge 2)$ be the n-dimensional Euclidean space and $\mathbb{S}^{n-1}$ denote the unit sphere in $\mathbb{R}^n$ equipped with induced Lebesgue measure $d\sigma$, and let $x' = \frac{x}{|x|}$ for any $x \neq 0$.

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